Acceleration of diffusion in randomly switching potential with supersymmetry

Alexander A. Dubkov[1,*] and Bernardo Spagnolo[2,†]
[1]*Radiophysics Department, Nizhni Novgorod State University, 23 Gagarin Ave., 603950 Nizhni Novgorod, Russia*
[2]*INFM-CNR, and Dipartimento di Fisica e Tecnologie Relative, Group of Interdisciplinary Physics, Università di Palermo, Viale delle Science, pad.18, I-90128 Palermo, Italy*

(Received 4 November 2004; revised manuscript received 22 July 2005; published 19 October 2005)

We investigate the overdamped Brownian motion in a supersymmetric periodic potential switched by Markovian dichotomous noise between two configurations. The two configurations differ from each other by a shift of one-half period. The calculation of the effective diffusion coefficient is reduced to the mean first passage time problem. We derive general equations to calculate the effective diffusion coefficient of Brownian particles moving in arbitrary supersymmetric potential. For the sawtooth potential, we obtain the exact expression for the effective diffusion coefficient, which is valid for the arbitrary mean rate of potential switchings and arbitrary intensity of white Gaussian noise. We find the acceleration of diffusion in comparison with the free diffusion case and a finite net diffusion in the absence of thermal noise. Such a potential could be used to enhance the diffusion over its free value by an appropriate choice of parameters.

DOI: 10.1103/PhysRevE.72.041104 PACS number(s): 05.40.−a, 02.50.−r, 05.10.Gg

I. INTRODUCTION

Brownian diffusion in periodic potential is an appropriate model to describe fluctuations of the Josephson supercurrent through a tunneling junction [1,2], diffusive motion of atoms in crystals [3], superionic conduction [4], charge density waves [5], chemical reactions [6], neuronal activity [7], synchronization of oscillations [8], rotating dipoles in external fields [9], particle separation by electrophoresis [10], and so on. Although qualitative asymptotic behavior of the system is well known, the calculations of particle mean drift, velocity spectrum, and the effective diffusion coefficient were performed in the general damped case only by simulations or by numerical solution of the Fokker-Planck equation [11]. Analytical results in the overdamped limit were obtained for the effective diffusion coefficient in arbitrary fixed periodic potential [12], for a symmetric periodic potential modulated by white Gaussian noise [13,14], for the mean velocity and the effective diffusion coefficient of a Brownian particle moving in a tilted periodic potential [15,16], and for the transition rates of Brownian motion with time-periodic driving force [17]. In the last case, as for modulation by white Gaussian noise [13,14], authors obtained the acceleration of diffusion in comparison with the free diffusion case.

The investigations of Brownian diffusion in fluctuating periodic potentials (flashing potential model) were also performed in the framework of the molecular motors problem, i.e., unidirectional motion of Brownian particles along the one-dimensional periodic structures [18]. As a rule, a two-state model in which asymmetric (ratchet-like) potential randomly switches between two different configurations was investigated [19], and in particular, the so-called *on-off* ratchet scheme was considered in Ref. [20]. The most important quantity to be calculated was the mean stationary flow of Brownian particles, but theoretical investigations were only performed for the simplest asymmetric potential as the sawtooth ratchet.

As it was recently shown, fluctuations of a potential via the random shifts of the one-half period provide a high efficiency with which a Brownian motor converts fluctuations into useful work [21]. These types of fluctuations can be caused by an external cyclic process generating the potential profile or a far-from-equilibrium chemical reaction resulting in a conformational change of the particle or of the track [22].

The sorting of Brownian particles by the enhancement of their effective diffusion coefficient is a subject of increasing scientific interest in the last years, both from experimental [23–26] and theoretical points of view [17,27]. Specifically, in Ref. [26], the interplay of global spatiotemporal symmetry and local dynamics was analyzed, and in Refs. [17,25], the enhancement of diffusion in *symmetric* potentials was investigated.

Motivated by these studies and by the importance of the problem of dopant diffusion acceleration in semiconductors for solid state microelectronics [28], we try to understand how nonequilibrium symmetrical correlated forces influence thermal systems when potentials are symmetric. This is done by using a randomly shifting potential satisfying the supersymmetry criterion [29]. We consider a different approach with respect to the previous theoretical investigations [21]. Starting from the analogy between a continuous Brownian diffusion at large times and the "jump diffusion" model [16], we reduce the calculation of effective diffusion coefficient to the first passage time problem. The general equations obtained are solved for the sawtooth periodic potential, and the exact expression for the effective diffusion coefficient is derived without any assumptions on the intensity of the white Gaussian noise and switchings mean rate of the potential. We find (i) the enhancement of diffusion as compared to free thermal diffusion, (ii) a finite net diffusion due to the potential fluctuations, and (iii) a diffusion independent on the thermal noise intensity for very deep potential wells.

*Email address: dubkov@rf.unn.ru
†Email address: spagnolo@unipa.it

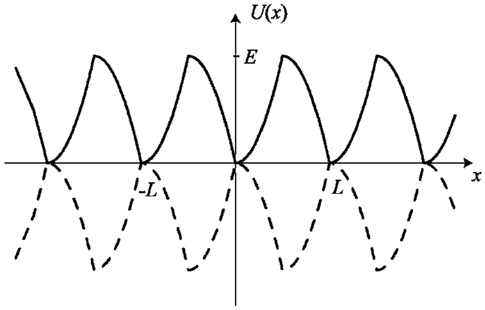

FIG. 1. Switching potential with supersymmetry.

II. GENERAL EQUATIONS

Let us consider the one-dimensional overdamped Brownian motion in randomly switching periodic potential $U(x)$

$$\frac{dx}{dt} = -\frac{dU(x)}{dx}\eta(t) + \xi(t), \tag{1}$$

where $x(t)$ is the Brownian particle displacement at time t, $\xi(t)$ is the white Gaussian noise with zero mean and intensity $2D$, $\eta(t)$ is a Markovian dichotomous noise, which takes the values ± 1 with switchings mean rate ν. Thus, we investigate the Brownian diffusion in a periodic potential flipping between two configurations, $U(x)$ and $-U(x)$ (see Fig. 1). In the "overturned" configuration, the maxima of the potential become the minima and vice versa. Further, we assume that the potential $U(x)$ satisfies the supersymmetry criterion [29]

$$E - U(x) = U\left(x - \frac{L}{2}\right), \tag{2}$$

where L is the spatial period of the potential (see Fig. 1). In accordance with Eq. (2), we can rewrite Eq. (1) as

$$\frac{dx}{dt} = -\frac{\partial}{\partial x}U\left(x + \frac{L}{4}[\eta(t) - 1]\right) + \xi(t), \tag{3}$$

i.e., as in Ref. [21], we consider the fluctuations of the potential via random shifts of one-half period $L/2$.

In such a situation, the ratchet effect is absent: $\langle \dot{x} \rangle = 0$, and following Ref. [12], we can determine the effective diffusion coefficient as the limit

$$D_{eff} = \lim_{t\to\infty} \frac{\langle x^2(t)\rangle}{2t}. \tag{4}$$

For a fixed periodic potential $[\eta(t)=1]$ the exact expression for D_{eff}, obtained in Ref. [12], has the form

$$\frac{D_{eff}}{D} = \left[\frac{1}{L}\int_0^L e^{U(x)/D}dx \frac{1}{L}\int_0^L e^{-U(x)/D}dx\right]^{-1}. \tag{5}$$

In the case when the modulation $\eta(t)$ is an additional white Gaussian noise, statistically independent of $\xi(t)$, with zero mean and intensity $2D_\eta$, the calculations give [13,14]

$$\frac{D_{eff}}{D} = \left[\frac{1}{L}\int_0^L \frac{dx}{\sqrt{1 + D_\eta [U'(x)]^2/D}}\right]^{-2}. \tag{6}$$

We place for convenience all Brownian particles at the origin at $t=0$. Because of the periodicity of the potential, the diffusion process can be coarsely conceived as consecutive transitions of the Brownian particle from the points of potential minima $x_m = mL$ to the nearest neighboring points $x_{m\pm 1}$. The transition time represents the escape time over left or right absorbing boundaries $x = x_{m\pm 1}$ for a particle starting from the point $x = x_m$, i.e., the random first passage time. Thus, we can consider, as in Ref. [15], the jumped diffusion model

$$\tilde{x}(t) = \sum_{k=1}^{n(0,t)} q_k, \tag{7}$$

where q_k are the random increments of a jump with values $\pm L$ and $n(0,t)$ denotes the total number of jumps in the time interval $(0,t)$. In the asymptotic limit $t\to\infty$, the random processes $x(t)$ and $\tilde{x}(t)$ become statistically equivalent, i.e., $\langle x^2(t)\rangle \simeq \langle \tilde{x}^2(t)\rangle$.

The non-Markovian random process $x(t)$ has Markovian dynamics between flippings, i.e., involves alternating pieces of two Markovian random processes $x_1(t)$ and $x_2(t)$, which are governed by Langevin equations [see Eq. (1)]

$$\dot{x}_1 = -U'(x_1) + \xi(t), \quad \dot{x}_2 = U'(x_2) + \xi(t).$$

The random increments q_i and the waiting times t_j between jumps are, therefore, statistically independent of each other and have the same probability densities, $W(q)$ and $w(t)$, respectively. Specifically the distribution of waiting times t_j is

$$w(t) = \frac{w_+(t) + w_-(t)}{2}, \tag{8}$$

where $w_+(t)$ and $w_-(t)$ are the first passage time distributions for the configuration of the potential with $\eta(0) = +1$ and for the initially overturned configuration $[\eta(0) = -1]$, respectively. Because of the symmetry of the potential $U(x)$ and of the dichotomous noise $\eta(t)$, the probabilities of transitions to the left and to the right have the same value, and the distribution $W(q)$ reads

$$W(q) = \frac{1}{2}[\delta(q - L) + \delta(q + L)]. \tag{9}$$

By calculating $\langle \tilde{x}^2(t)\rangle$ from Eqs. (7)–(9) and substituting in Eq. (4), we can express the effective diffusion coefficient in terms of the mean waiting time (see, for example, Refs. [14,15])

$$D_{eff} = \frac{L^2}{2\tau}. \tag{10}$$

In accordance with Eq. (8), τ is the semisum of the mean first passage times (MFPTs) τ_+ and τ_-, corresponding to the probability distributions $w_+(\tau)$ and $w_-(\tau)$.

For our system (1), the exact equations for the MFPTs for Brownian diffusion in randomly switching potentials, derived from the backward Fokker-Planck equation [30], are

$$D\tau_+'' - U'(x)\tau_+' + \nu(\tau_- - \tau_+) = -1,$$

$$D\tau_-'' + U'(x)\tau_-' + \nu(\tau_+ - \tau_-) = -1, \qquad (11)$$

where $\tau_+(x)$ and $\tau_-(x)$ are the MFPTs for initial values $\eta(0)=+1$ and $\eta(0)=-1$, respectively, with the starting position of Brownian particles at the point x. We consider the initial position at $x=0$ and solve Eqs. (11) with absorbing boundaries at $x=\pm L$

$$\tau_\pm(-L) = 0, \quad \tau_\pm(L) = 0. \qquad (12)$$

Then by substituting $\tau=[\tau_+(0)+\tau_-(0)]/2$ in Eq. (10), we find

$$D_{eff} = \frac{L^2}{\tau_+(0) + \tau_-(0)}. \qquad (13)$$

We place a reflecting boundary at the origin, because the probability flow at the point $x=0$ equals zero for any times. We can solve, therefore, the set of equations (11) in the range $(0,L)$ with the following boundary conditions (see Ref. [30]):

$$\tau_\pm'(0) = 0, \quad \tau_\pm(L) = 0, \qquad (14)$$

which are equivalent to Eqs. (12). By introducing two new auxiliary functions

$$T(x) = \frac{\tau_+(x) + \tau_-(x)}{2}, \quad \theta(x) = \frac{\tau_+(x) - \tau_-(x)}{2}, \qquad (15)$$

we can rewrite Eqs. (11) as

$$R' + f(x)\theta' = -\frac{1}{D},$$

$$\theta'' + f(x)R - \frac{2\nu}{D}\theta = 0, \qquad (16)$$

where $R(x)=T'(x)$ and $f(x)=-U'(x)/D$. According to Eqs. (14) and (15), we have the following boundary conditions for Eqs. (16):

$$R(0) = \theta'(0) = 0, \quad T(L) = \theta(L) = 0. \qquad (17)$$

After integrating the first equation (16) in the interval $(0,x)$ and using the boundary condition (17) for $R(0)$, we get

$$R(x) = -\frac{x}{D} - \int_0^x f(y)\theta'(y)dy. \qquad (18)$$

By substituting $R(x)$ into the second Eq. (16), we obtain the following integro-differential equation for the function $\theta(x)$

$$\theta'' - f(x)\int_0^x f(y)\theta'(y)dy - \frac{2\nu}{D}\theta = \frac{xf(x)}{D}. \qquad (19)$$

To find the value $T(0)$, we integrate Eq. (18) in the interval $(0,L)$

$$T(L) = -\frac{L^2}{2D} - \int_0^L (L-x)f(x)\theta'(x)dx + T(0). \qquad (20)$$

Using the boundary conditions (17) and substituting $T(0)$ in Eq. (13), we find finally

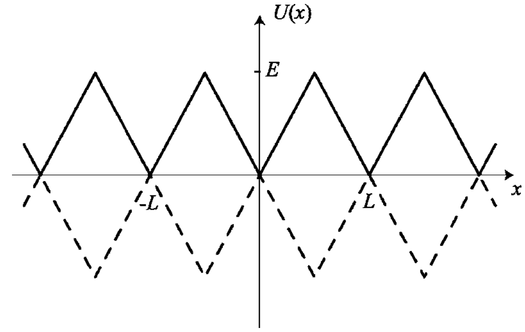

FIG. 2. Switching sawtooth periodic potential.

$$\frac{D_{eff}}{D} = \left[1 + \frac{2D}{L}\int_0^L \left(1 - \frac{x}{L}\right) f(x)\theta'(x)dx\right]^{-1}. \qquad (21)$$

Equations (19) and (21) solve formally the problem. The sign of the integral term in Eq. (21) determines the acceleration or slowing down of diffusion in comparison with the case when particles diffuse freely. The sign "+" corresponds to the slowing down and the sign "−" corresponds to the acceleration. Unfortunately, Eq. (19) cannot be solved for arbitrary periodic potential $U(x)$. Further, we analyze Eqs. (19) and (21) for the sawtooth periodic potential and derive, for this potential, the exact formula for the effective diffusion coefficient D_{eff}.

III. SAWTOOTH PERIODIC POTENTIAL

Let us consider the symmetric sawtooth periodic potential

$$U(x) = \begin{cases} kx, & 0 \leq x \leq L/2 \\ k(L-x), & L/2 \leq x \leq L \end{cases}, \qquad (22)$$

where $k=2E/L$ (see Fig. 2). We solve Eq. (19) for the regions $0<x<L/2$ and $L/2<x<L$ separately, using the boundary conditions (17)

$$\theta'(0) = 0, \quad \theta(L) = 0, \qquad (23)$$

and the continuity conditions for functions $\theta'(x)$ and $\theta(x)$ at the point $x=L/2$

$$\theta'\left(\frac{L}{2} - 0\right) = \theta'\left(\frac{L}{2} + 0\right), \quad \theta\left(\frac{L}{2} - 0\right) = \theta\left(\frac{L}{2} + 0\right). \qquad (24)$$

To obtain the solution in interval $(0,L/2)$, we substitute $f(x)=-k/D$ in Eq. (19) and we get

$$\theta'' - \gamma^2\theta = -\frac{kx}{D^2} - \frac{k^2}{D^2}\theta(0), \qquad (25)$$

where $\gamma=\sqrt{k^2/D^2+2\nu/D}$. The general solution of Eq. (25), which satisfies the first boundary condition (23) reads

$$\theta(x) = c_1\left(\cosh \gamma x + \frac{k^2}{2\nu D}\right) - \frac{k}{\Gamma^3}(D \sinh \gamma x - \Gamma x),$$

$$\theta'(x) = c_1\frac{\Gamma}{D} \sinh \gamma x - \frac{k}{\Gamma^2}(\cosh \gamma x - 1), \tag{26}$$

where $\Gamma = \gamma D$.

To solve Eq. (19) in the range $L/2 < x < L$, we write it as

$$\theta'' + f(x)\int_x^L f(y)\theta'(y)dy - \frac{2\nu}{D}\theta = \frac{xf(x)}{D} + f(x)\int_0^L f(y)\theta'(y)dy.$$

Substituting in this equation $f(x) = k/D$ for interval $(L/2, L)$ and taking into account the second boundary condition (23), we obtain

$$\theta'' - \gamma^2\theta = \frac{kx}{D^2} + \frac{k}{D}A, \tag{27}$$

where we introduced an additional unknown constant

$$A = \int_0^L f(x)\theta'(x)dx. \tag{28}$$

The general solution of Eq. (27) can be written as

$$\theta(x) = c_2 \cosh \gamma(x-L) + c_3 \sinh \gamma(x-L) - \frac{kx}{\Gamma^2} - \frac{kD}{\Gamma^2}A. \tag{29}$$

If we put $x = L$ in Eq. (29) and use the second boundary condition (23), we arrive at

$$c_2 = \frac{k}{\Gamma^2}(L + DA), \tag{30}$$

and, as a consequence,

$$\theta(x) = c_2[\cosh \gamma(x-L) - 1] + c_3 \sinh \gamma(x-L) + \frac{k(L-x)}{\Gamma^2},$$

$$\theta'(x) = \frac{\Gamma}{D}[c_2 \sinh \gamma(x-L) + c_3 \cosh \gamma(x-L)] - \frac{k}{\Gamma^2}. \tag{31}$$

Using the solutions (26) and (31), continuity conditions (24), and Eqs. (28) and (30), it is easily to obtain the following algebraic set of equations for unknown constants c_1, c_2, and c_3

$$c_1 \sinh \alpha + c_2 \sinh \alpha - c_3 \cosh \alpha = \frac{kD}{\Gamma^3}(\cosh \alpha - 2),$$

$$c_1\left(\cosh \alpha + \frac{k^2}{2\nu D}\right) - c_2(\cosh \alpha - 1) + c_3 \sinh \alpha = \frac{kD}{\Gamma^3} \sinh \alpha,$$

$$c_1\left(2 \cosh \alpha - 1 + \frac{k^2}{2\nu D}\right) + \frac{\Gamma^2}{k^2}c_2 = \frac{2\nu DL}{k\Gamma^2} + \frac{2kD}{\Gamma^3} \sinh \alpha, \tag{32}$$

where $\alpha = \gamma L/2$. Substituting Eqs. (26) and (31) in Eq. (21) and performing the integration, we find

$$\frac{D_{eff}}{D} = \left\{1 + \frac{2k}{L}\left[-\left(c_1 + \frac{c_3}{2\alpha} - \frac{kD}{2\alpha\Gamma^3}\right)(\cosh \alpha - 1) + \frac{1}{2}\left(c_2 - c_1 + \frac{2kD\alpha}{\Gamma^3}\right)\left(\frac{\sinh \alpha}{\alpha} - 1\right) - c_1\frac{\Gamma^2}{4\nu D}\right]\right\}^{-1}. \tag{33}$$

The solutions of the set of Eqs. (32) are

$$c_1 = \frac{4kD\mu^2 \sinh \alpha - \alpha(\sinh^2 \alpha/2)}{\Gamma^3(1 - 2\mu + 4\mu \cosh \alpha + \mu^2 \cosh 2\alpha)},$$

$$c_2 = \frac{2kD[\mu\alpha(\cosh \alpha + \mu \cosh 2\alpha) + (1 - 2\mu + 3\mu \cosh \alpha)\sinh \alpha]}{\Gamma^3(1 - 2\mu + 4\mu \cosh \alpha + \mu^2 \cosh 2\alpha)},$$

$$c_3 = \frac{kD[7\mu - 1 - \mu^2 + 2(1 - 4\mu + \mu^2)\cosh \alpha + 3\mu \cosh 2\alpha + 2\mu\alpha(1 - \mu + 2\mu \cosh \alpha)\sinh \alpha]}{\Gamma^3(1 - 2\mu + 4\mu \cosh \alpha + \mu^2 \cosh 2\alpha)}, \tag{34}$$

where we introduced the new dimensionless parameter $\mu = 2\nu D/k^2$. Substituting expressions (34) in Eq. (33), we obtain finally

$$\frac{D_{eff}}{D} = \frac{2\alpha^2(1+\mu)(1 - 2\mu + 4\mu \cosh \alpha + \mu^2 \cosh 2\alpha)}{2\alpha^2\mu^2(1+\mu) + 2\mu(7 - \mu + 2\alpha^2\mu^2)\sinh^2 \alpha + 4\alpha\mu(1 - 3\mu + 4\mu \cosh \alpha)\sinh \alpha + 8(1 - 6\mu + \mu^2)\sinh^2(\alpha/2)}. \tag{35}$$

It must be emphasized that the result (35), for the effective diffusion coefficient of Brownian particles in switching sawtooth periodic potential, was derived without any assumptions on the intensity of white Gaussian noise, the mean rate of switchings, and the values of the potential profile parameters.

IV. ACCELERATION OF DIFFUSION

Let us analyze the possibility to accelerate a diffusion in the flipping sawtooth periodic potential in comparison with a free diffusion.

First of all, we introduce two new dimensionless parameters having a clear physical meaning

$$\beta = \frac{E}{D}, \quad \omega = \frac{\nu L^2}{2D}. \tag{36}$$

The parameter β is the ratio between the height of the potential barrier and the intensity of white Gaussian noise. The parameter ω is the ratio between the time of a free diffusion through the distance L and the mean time interval between switchings. The dimensionless parameters α and μ, involved in Eq. (35) for the effective diffusion coefficient, can be expressed in terms of β and ω as

$$\alpha = \sqrt{\beta^2 + \omega}, \quad \mu = \frac{\omega}{\beta^2}. \tag{37}$$

Let us investigate the limiting cases of very small and very large parameters β and ω. At very rare flippings $(\omega \to 0)$ we have, in accordance with Eq. (37), $\alpha \simeq \beta$, $\mu \to 0$, and Eq. (35) gives

$$\frac{D_{eff}}{D} \simeq \frac{\beta^2}{4\sinh^2(\beta/2)}, \tag{38}$$

which coincides with the result obtained from Eq. (5) for the potential (22). Because of $\sinh x > x (x>0)$, we have $D_{eff} < D$, and diffusion slows down in the fixed periodic potential in comparison with the case when Brownian particles diffuse freely.

In the opposite case of very fast switchings $(\omega \to \infty)$, we can predict the result. In such a situation, the Brownian particles "see" the average potential i.e., $[U(x)+(-U(x))]/2=0$, and, as a result, we obtain diffusion in the absence of a potential. Actually, if we put in Eq. (35), $\alpha \simeq \sqrt{\omega}[1+\beta^2/(2\omega)] \to \infty$ and $\mu = \omega/\beta^2 \to \infty$, we find

$$\frac{D_{eff}}{D} \simeq 1 + \frac{\beta^2}{\omega}. \tag{39}$$

As Eq. (39) indicates, by fast switching of a potential, we always obtain the acceleration of diffusion in comparison with a free diffusion case.

From the exact expression (35), we plot in Fig. 3 the normalized effective diffusion coefficient D_{eff}/D as a function of the dimensionless mean rate of potential switching ω, for different values of the dimensionless height of potential barriers β. We have a nonmonotonic behavior for all values β. The rate of diffusion becomes greater than the rate of a free diffusion above some value of ω. This value decreases with the increasing height of the potential barrier.

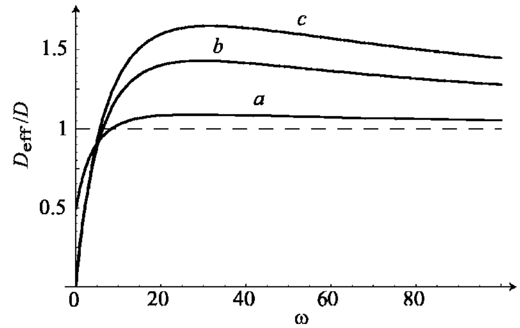

FIG. 3. The normalized effective diffusion coefficient versus the dimensionless switchings mean rate of potential $\omega = \nu L^2/(2D)$ for different values of the dimensionless height of the potential barrier. Namely $\beta = 3, 7, 9$, for the curves a, b, and c, respectively.

In the limiting case of $\beta \ll 1$, we find from Eq. (35),

$$\frac{D_{eff}}{D} \simeq 1 + \frac{\beta^2}{2\omega^2 \cosh 2\sqrt{\omega}}[(1+2\omega)\cosh 2\sqrt{\omega} - (4\cosh\sqrt{\omega} - 3)(1 + 4\sqrt{\omega}\sinh\sqrt{\omega} - 2\omega)]. \tag{40}$$

As the analysis of Eq. (40) shows, for relatively low barriers, we obtain the enhancement of diffusion just at relatively fast switchings: $\omega > 9.195$.

For very high potential barriers $(\beta \to \infty)$ and fixed mean rate of switchings ν, we have, from Eq. (37), $\alpha \simeq \beta \to \infty, \mu \to 0, \alpha^2\mu \to \omega$. As a result, we find from Eq. (35),

$$\frac{D_{eff}}{D} = \frac{2}{7}\omega \tag{41}$$

or, in accordance with Eq. (36),

$$D_{eff} = \frac{\nu L^2}{7}. \tag{42}$$

We obtained an interesting result: a diffusion at superhigh potential barriers (or at very deep potential wells) is due to the switchings of a potential only. According to Eq. (42), the effective diffusion coefficient depends on the mean rate of flippings and the spatial period of potential profile only and does not depend on D. In the case of an asymmetric potential with a very high barrier to one side, this mechanism without diffusive steps provides the high efficiency of the Brownian motor, because this barrier blocks the counterflow of particles [21].

Let us explain this result (42). A diffusion is practically absent at very rare switchings $(\nu \to 0)$, because the Brownian particles are not able to cross such high potential barriers. They can move in both directions by flippings only. After the first switching, the Brownian particles fall rapidly in the nearest potential wells at the points $x = \pm L/2$ and then wait for the next potential switching. As a result, from Eq. (10) (with $L/2$ instead of L), we obtain $D_{eff} \simeq L^2/(8\langle\tau\rangle)$, where

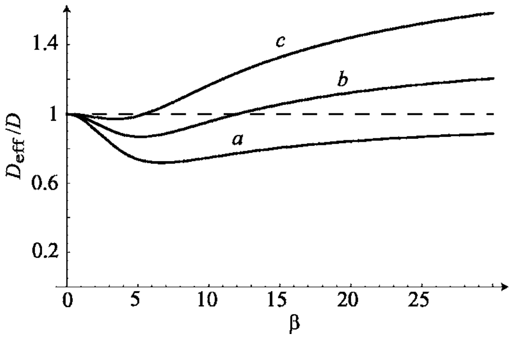

FIG. 4. The normalized effective diffusion coefficient versus the dimensionless height of potential barriers $\beta=E/D$ for different values of the dimensionless switchings mean rate of the potential. Namely $\omega=3.5,5,7$ for the curves a, b, and c, respectively.

$\langle\tau\rangle=1/\nu$. Thus, the smaller the mean time interval between flippings $\langle\tau\rangle$ is or the greater the spatial period L is the greater the rate of diffusion is.

The behavior of the normalized effective diffusion coefficient D_{eff}/D as a function of the dimensionless height of the potential barrier β for three values of the dimensionless mean rate of switchings ω is shown in Fig. 4. We note that all curves are nonmonotonic. The rate of diffusion decreases for small values of β [see Eq. (40)], reaches a minimum, and then tends to the asymptotic value given by Eq. (42). We observe the acceleration of diffusion for any β just for parameter $\omega>9.195$.

The area of diffusion acceleration, obtained by Eq. (35), is shown on the plane (β,ω) in Fig. 5 as the shaded area.

This area lies inside the rectangular region $\beta>0$ and $\omega>3.5$. The three-dimensional (3D) plot of D_{eff}/D as a function of β and ω is shown in Fig. 6.

Finally, we can compare our result (35) with the case of modulation by external white Gaussian noise [see Eq. (6)]. We formally introduce the amplitude a of a Markovian dichotomous noise $\eta(t)$ by replacing β with $a\beta$ and then by making the limit $a\to\infty$, $\nu\to\infty$, and $a^2/\nu\to 2D_\eta$. This is because the Markovian dichotomous noise becomes, in this limit, the white Gaussian noise with intensity $2D_\eta$. From Eq. (35), therefore, we get

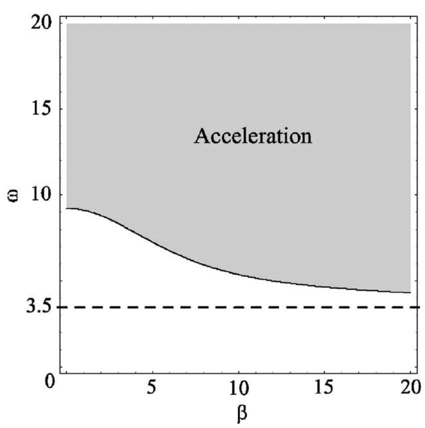

FIG. 5. The shaded area is the parameter region on the plane (β,ω) where the diffusion acceleration compared with a free diffusion case can be observed. Here $\beta=E/D$ and $\omega=\nu L^2/(2D)$.

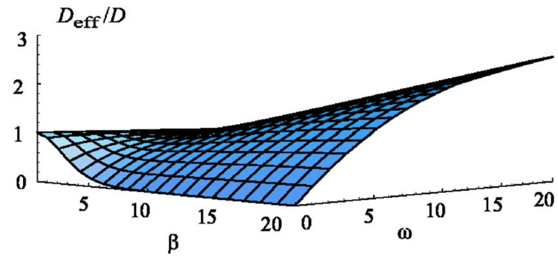

FIG. 6. (Color online) 3D plot of normalized effective diffusion coefficient versus the relative height of potential barriers β and the dimensionless mean rate of potential flippings ω.

$$D_{eff}=D+k^2D_\eta. \tag{43}$$

Equation (43) coincides with the formula previously obtained in Refs. [13,14], which is derived by substituting the potential profile (22) in Eq. (6).

To analyze the dependence of the effective diffusion coefficient D_{eff} on the friction coefficient h, corresponding to the situation in which different types of particles are moving in the same periodic potential, we replace in Eqs. (35)–(37): E with E/h and D with D/h. This is because we put in Eq. (1) $h=1$. In Fig. 7, we report the behaviors of the normalized effective diffusion coefficient D_{eff}/D as a function of the friction coefficient h for different values of dimensionless parameters β and ω. The nonmonotonic behavior of the normalized effective diffusion coefficient shown in Fig. 7 gives evidence that the dynamics of Eq. (1) can act as a device for

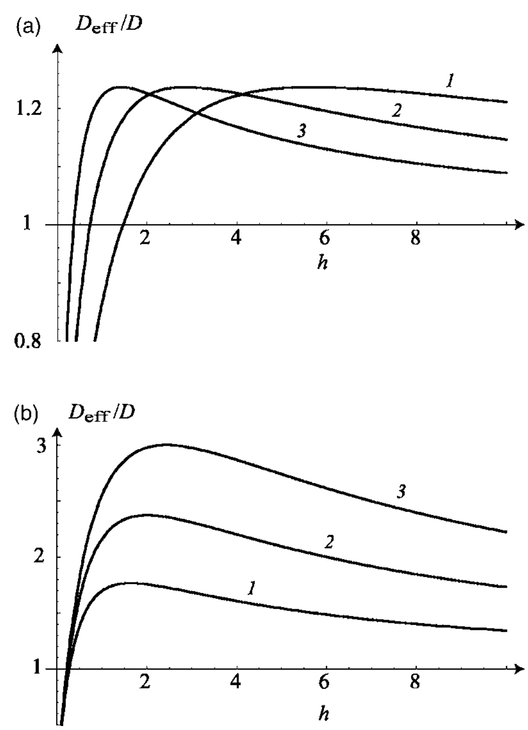

FIG. 7. (a) The normalized effective diffusion coefficient as a function of the friction coefficient h for fixed $\beta=5$ and different values of ω: 5, 10, 20. (b) The normalized effective diffusion coefficient as a function of the friction coefficient h for fixed $\omega=20$ and different values of β: 10, 15, 20.

sorting different types of particles by enhancement of the diffusion process. Therefore, according to the Stokes' law on the friction coefficient, particles with small size will be accelerated more than particles with greater size at the high switchings mean rate of the potential. While vice versa at low switchings rate of the potential, Brownian particles with great size will be accelerated more with respect to particles with smaller size. The maximum of D_{eff}/D shifts toward higher values of the friction coefficient as the frequency ω decreases, for a fixed value of the barrier height [Fig. 7(a)]. In Fig. 7(b), we report the enchancement of diffusion for fixed value of the parameter ω, namely $\omega=10$, and different values of the β parameter. The maximum of D_{eff}/D increases with increasing height of the barrier. This surprising effect is due to the high slope of the potential for high barriers.

V. CONCLUSIONS

We studied overdamped Brownian motion in a supersymmetric periodic potential switched by Markovian dichotomous noise between two opposite configurations. We reduced the problem to the mean first passage time problem and derived the general equations to calculate the effective diffusion coefficient. For the sawtooth potential with supersymmetry, we obtain the exact formula for the effective diffusion coefficient, which is valid for arbitrary intensity of white Gaussian noise and arbitrary parameters of the external dichotomous noise and of potential. We obtained the area on the parameter plane (β,ω) where the acceleration of diffusion can be observed. We analyzed in detail the limiting cases of very high and very low potential barriers, as well as very rare and very fast switchings. A diffusion process is obtained in the absence of thermal noise. Supersymmetric periodic potentials can be used to enhance the diffusion over its free value by an appropriate choice of parameters. Our results could be used to sort particles by their friction coefficient or by their size. The interesting experiment reported in Ref. [24] showed that it is possible to obtain the experimental setup with the desired properties of symmetry and periodicity of the potential structure. Finally, another important application of our results could be to speed up the diffusion process of dopant in the *p*-*n* junction in heterostructures [28].

ACKNOWLEDGMENTS

This work has been supported by MIUR, INFM-CNR, Russian Foundation for Basic Research (Project No. 05-02-16405), and by the program, "Leading Scientific Schools of Russia" (Project No. 1729.2003.2).

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